

1-2018

## Mathematics education (research) liberated from teaching and learning: Towards (the future of) doing mathematics

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### Recommended Citation

Maheux, Jean-Francois and Proulx, Jerome (2018) "Mathematics education (research) liberated from teaching and learning: Towards (the future of) doing mathematics," *The Mathematics Enthusiast*: Vol. 15 : No. 1 , Article 6.

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## Introduction



Welcome. Welcome in. Please take a seat. Or actually don't! You'd probably better read this on your feet. That's how we wrote it anyways...

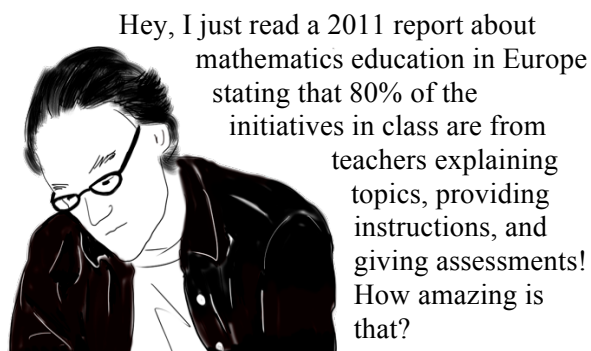
So let's get started, shall we? The title of this piece is the following:

### **Mathematics education (research) liberated from teaching and learning: Towards (the future of) doing|mathematics**

Now, what do you make of that? We want to discuss mathematics education, and research in mathematics education (we don't really need to specify, hence the parenthesis). And we want to address teaching and learning (mathematics, but we skipped the parenthesis this time around because we used the word just before) in terms of *not-being*. You will notice, however, the provocative presence of "liberated", a strong verb which alludes to the idea of freedom while acknowledging certain constraints: captivity, dependence, and even liability. Its selection was a bold choice. But having made such a suggestive statement (without actually saying things upfront), we hastily print a colon to bring in a potential alternative – more positive, one should hope. This alternative mentions to be about going "towards" something. If we care enough to read the next parenthesis (yep, another set!), we realize that this something does not exist yet, since it lies in the future. This something we finally name with the title's last breath: "doing|mathematics". For those who haven't read our recent articles (e.g., in French, Maheux & Proulx, 2014, or in English, Maheux & Proulx, 2015), what *that* is will remain obscure for some time. In a certain way, it is also still mysterious for us: it is the object of our *research*, so we haven't "found" it yet. We haven't really found out what "doing|mathematics" is, or what it does. As a piece of research, this article thus also aims to help *us* figure some of these things out. As a result, this communication piece is not merely for you. And after all, as Von Foerster used to say, in the end you'll know more about us than about the topic of the paper.

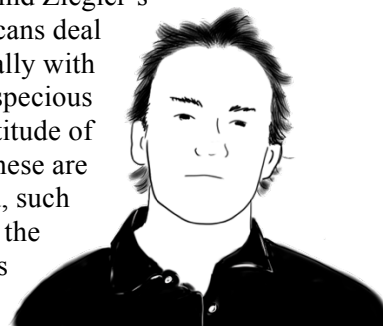
Still with us? Great!

That was honestly the toughest part of the paper. But wait... it's not over yet!



Hey, I just read a 2011 report about mathematics education in Europe stating that 80% of the initiatives in class are from teachers explaining topics, providing instructions, and giving assessments! How amazing is that?

Sure... And let's remind Ziegler's "Some 82.49% of Americans deal much too uncritically with numbers; they fall for the specious authority and exactitude of numbers – even when these are sometimes totally invented, such as the percentage at the beginning of this sentence."!



## Mathematics Education (Research)

JEROME: So, what do you want to do with this paper? How do we start? I've been teaching (!) about the uselessness of teaching for quite a while now, but I've never had the guts to write on it, mainly scared that I would be required to do a huge background check on the history, develop theoretical foundations, etc., whereas it is for me mainly an idea, a visceral one. And, I'm not talking about teaching as "craft knowledge" or some other thing, as some say... it is like an orientation. Sometimes I'd love to scream it loud and clear! But eh, maybe we can just do that in this Special Issue?

JF: Yes, yes, I know. (Whoa! You start strong!) The title suggests that thinking about mathematics education (research) in terms of teaching and learning seems unavoidable. We could easily talk about the origin of the discipline. Go back to Klein for example, and his observations on a “double discontinuity” between school and university (what is taught, what is learned). There is something lax about wanting to fix things without really questioning what or why they are, and without imagining radically different premises...

JEROME: The old recipe of trying to improve a bad idea! But is it tricky to go back to the “past”, with Klein for example, in order to assert an idea. You know Klein is not really saying anything about the necessity to think in terms of teaching and learning. But I can see how his orientation in terms of content is supportive of this kind of organization and way of thinking. I guess Binet (1899) with his essay in the first volume of *l'Enseignement Mathématique* does something similar, wanting to scientifically find the best pedagogical practices so that children will demonstrate understanding of selected concepts, a bit like Brousseau or Balacheff...

For a view on our scientific rant against content & knowledge, (re-)read our Maheux & Proulx (2014, 2015).

For a view on our scientific rant against content & knowledge, (re-)read our Maheux & Proulx (2014, 2015).

JF: Exactly. So the idea is not to pretend that anybody said we *should* be thinking about mathematics education in terms of teaching and learning, but rather to demonstrate through a series of examples that this is a very general and maybe unquestioned way of thinking about mathematics education.

JEROME: Ok, better than simply shouting out loud. We should keep it short, I mean, people will have intuited this already. But many will probably want to disagree, and insist that we've always had individuals thinking outside of that paradigm. Take Montessori for example: a large part of her work is concerned with *providing children with a rich environment* to aid their natural development. Or think of Freinet with his recurrent analogy about becoming able to walk and to talk. Where's the "teaching"?

JF: Even when they write about the educators “demonstrating” things so children can imitate them, we have the feeling that it is more a question of exposing children to ways of doing, rather than teaching them stuff to learn. Here again this offering of a “*culturally-rich*” environment, as Papert would say. But you’ll agree that this is still mainly viewed as “alternative” education in the face of what, in the US for example, the NCTM is all about, and has been since 1920! Plus all the stuff that cumulate in and came out of the math wars, and so on.

JEROME: And in research too. It might feel like the vast majority of the research work in mathematics education revolves (in various ways, at different distances) around questions of teaching and learning, but this is certainly *not* the only center of attraction. At least not for us...

JF: But it is still interesting to conceptualize such an orientation and the opportunity to turn away from it. In our own studies over the last few years, we adopt a position that does not situate teaching and learning as a central element in mathematics education (research).

JEROME: OK but... we need some nuances. Not everybody means the same thing when they use these words, and we don't want people to think we are calling them idiots for their phrasing.

JF: Of course not! But two things. First: Yes, there are many ways to think about teaching and learning and even some very clear disagreements about what these words mean (take for example Vygotsky's Russian word *obuchenie*, which means both teaching and learning!). But it is clear to me that behind those nuances, we can still find some common ground: the presence of an agenda, an interest in changing "learners", etc.

JEROME: Oh, yes, this desire to change things...to change the (bad/wrong/incomplete) other... some Gandhi, please!!

JF: ... and this recurrent assumption of a distinction in nature between what teachers and students do.

JEROME: You say it's a bad thing? Isn't there something that says there is nothing more unequal than treating everybody equally?

JF: Oh oh! You know I like playing with words. Doing "the same thing" does not mean doing 'it' in an identical manner! But let's not get political here. You hate that anyway. I'm just saying that thinking in terms of teaching and learning comes with certain kinds of problems. I want to call it a paradigm. What comes in the next section with Nietzsche and Derrida aims at illustrating *some* of the problematic issues related to this (somehow archaic) paradigm.

JEROME: Let's still clarify a point right now: we are not saying we want to get rid of teachers and schools. Again, with Papert's idea of culturally-rich environments... the Samba schools...

JF: That was my second point. Illich's ideas are interesting, but that's not the point. Changing paradigm *might* lead us to want to get rid of our schooling systems, or transform them profoundly... But, for some people so does the current paradigm, e.g., in the "humanist" tradition. However, I don't see it as the only option. And more importantly: this article is not about that. Otherwise the title would be "Liberated from Teachers and Schools", or something!

JEROME: Good point. I think clarifying the focus can make the invitation to change paradigm more engaging. Now what about research?

JF: Tell me if this is still unclear... The focus on "research" goes in the same direction. We are talking paradigms, ways of thinking, and for me that's the stuff of research. Again, the title is not "Mathematics Education (Institutions)" for example. The idea here is to present a potential paradigm shift regarding mathematics education, and the stuff that goes on when people do mathematics in school, as a *research* perspective (and as the result of a research endeavor).

JEROME: It is something to research, or to research with.

JF: In the same way Newton introduced gravity as a radically new way to think about bodies in motion! He didn't say "this is how gravity works", let alone "this is how gravity should be working". Rather, he offered to *examine motion in those terms*. People ended up adopting his theory, but not in a purely passive way. The notion of action at distance always bothered physicists, leading to such variations as the space-time curvature model, or the recently publicized observation of gravitons.

JEROME: Newton! Sounds a little bit pretentious... I feel my recent 2015 piece in *For the Learning of Mathematics* said the same thing about seeing mathematics education research as study... We don't need Newton.

JF: Hahaha! We sure don't! I hope readers won't forget the playful, humorous aspect of all this, of what we do here. This is 'serious work', but "destabilizing taken-for-granted truths that lay at the

heart of mathematics education as a research field” is also for me breaking with seriousness. So let’s have some fun.

JEROME: As you know, research for me is about offering distinctions, ones that aim to be useful. Truth is what works! This is also liberating!

JF: You and your usefulness... I’ll stick to the fun! Starting with Nietzsche and his critique around the impossibility of truly “revealing” pupils to themselves through institutional teaching. Then we can consider Derrida’s analysis of how a teaching/er’s body and a body of knowledge are in constant conflict, erasing one another.

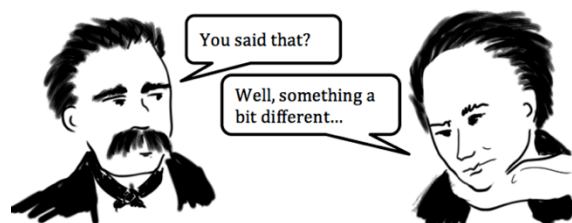
JEROME: Oh...but...I can’t help noticing that we haven’t talked about mathematics...

JF: But this is our doing|mathematics part!

JEROME: Yes, but still. Even though I enjoy flirting with philosophical ideas, I am a didactician of mathematics – and all that comes with that – like it or not!

JF: Let’s just hold your horses. We don’t want to bring in too much stuff too quickly. And our eccentric views about mathematics can also scare some readers off. We want them to read a bit more!

JEROME: Ok, later then...



## Liberated from Teaching and Learning

**H**ow can mathematics education be conceived of outside of teaching and learning, and why might we want to do so? Answering the first question is relatively easy. There are many ways to conceptualize educational matters without explicitly referencing (or defining) teachers and students. For example, we don’t generally think about infants as students, or even as learners, even though it is clear that some mathematical “development” seems to take place from the earliest age. We can also think about situated learning for example, in

Infants as students? Hey, all is possible... Check out Cockburn’s (1986) dissertation!

Are researchers considered as “students” who are “learning” about the things they study? Probably not...

which *participation* is the driving concept. In situated learning, for example, apprenticeship is not in essence about teaching and learning. Rather, apprentice focuses on developing expertise and social recognition through actual practice. Insights gained by the research with/in such different frameworks often transpired in general educational discourse. The idea of a “social-

We could quote from Lave and Wenger (1991) when they explicitly say they are not talking about learning and schools.

constructivism” might be an example of such thinking. Hence, the philosophical work providing the foundation of these ways of thinking about human experience, what came to be called radical constructivism, has little to do with the project of having students discover rules for adding numbers in the fractional form, or the constitution of “taken-as-shared” classroom mathematical norms. Nevertheless... for now, dear reader, let’s hear what Nietzsche and Derrida have to say.



Nietzsche, Derrida

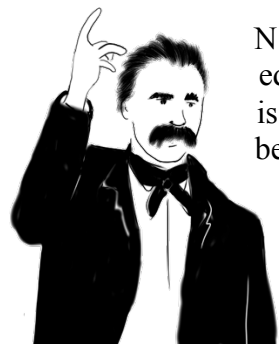
N: I\* am saying that what is done in schools and even at the university has nothing to do with the true nature of education: the formation of the most desirable self. Education comes from the Latin

Gradgrind: “Teach these boys and girls nothing but facts. Plant nothing else, and root out everything else. You can only form the minds of reasoning animals upon Facts; nothing else will ever be of any service to them.”  
*Hard Times*, Charles Dickens

*educere* which mean “to lead out of” or “away from”. Those institutions do the opposite. They instruct people, preparing them to be well-fitted citizens. True education is not about knowing facts or understanding a particular stock of knowledge, or attaining cultural signifiers, or acquiring practical skills. This is what you call teaching (“to show”) and learning (“to follow a track”). Educating is an exhortation to overcome the inertia of tradition and conventionality, and become responsible for creating our own existence.

\*Authors’ note: We are in a way abusing Nietzsche here (and then others as well), but who cares? Most of these people are dead anyways! To put it differently, we take inspiration from what they wrote, focusing on aspects of their thinking which make sense in relation to what we want the reader to engage with. Is this not the pursuit at the very heart of all research, no matter what stuff we are looking at? Yes, Nietzsche for instance used the German words equivalent to “teaching”. We could argue that his use of the term redefines it altogether. And so we should talk of an N-teacher to avoid conflating what he writes with the usual image of what a teacher is. Or we could change the guy’s name to Mitch and make him say whatever we want. What matters to us is working with ideas. Hinting at where they are coming from is, foremost, a way to acknowledge where *we* are coming from: we are not delving in the past, we are reaching toward the future... Again, this is about *us*...and the reading you make of it is about...*you*!

D: Yes and no. Because, you see, this “teaching” phenomena you are talking about is an impossibility. You might think that there is a teacher passing on knowledge or even just information to be learned, but such passing on is impossible. The teacher is always, inevitably, in the way of what must be learned. Between the body of knowledge and those of the students, there is always a “teaching body” in two senses. There is a whole group of people who decide what is to be taught (a body of a complex and changing anatomy). And there is the person who actually (and bodily) stands (for this group) in the room. We cannot assume that the teaching double-body can erase itself. But that supposed neutrality is what constructs the very idea of teaching.



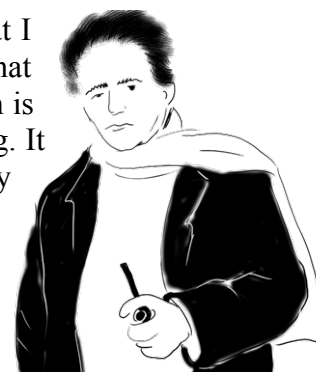
N: So we agree! I am saying that true *education* cannot be institutionalized. The true educator is the one who liberates you from the institution, and does that in a way that is absolutely unique to you (and to him/her), to make you go beyond yourself, and beyond anything he/she could present. Joy is another fundamental condition. The true

educator doesn’t perform a function, but acts out of joy. He/she honestly and joyfully talks and writes and does whatever he/she does first of all for him/herself. The true mathematics educator would be someone who truly does mathematics and whose

joy in it makes others joyful. It is *through* this educator’s powerful example that one can educate him/herself, reaching out for the preservation and the creation of culture, and striving thereby for the completion of Nature. In this way there are, in fact, no educators: we should speak only of self-education.

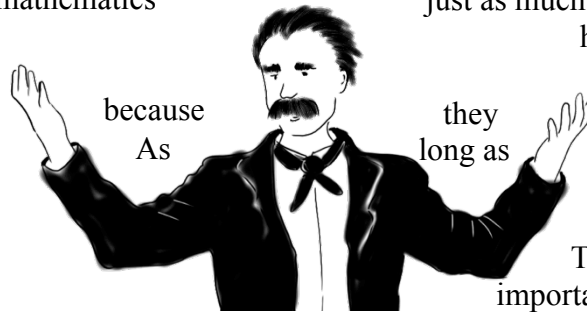
And so the teacher too needs to enjoy the educational process. Education is not uniquely oriented toward students. Teachers count too!

D: What I am saying is slightly different, but similar. Education for me is what I have always been writing and talking about: a deconstructive process, for what they call that. It is the active work of doing what you do. In this case, education is mathematics in the making, in the process of becoming, in the moment of being. It is deconstruction because it rests on educators giving up on what they think they know to genuinely engage with the unknown (e.g., really *do* mathematics). Otherwise, children will keep on going to school simply out of obligation, with teachers lifelessly formatting kids for a faceless, bodyless world. So yes, it is the task of the educator to release curiosity, spark off desire, give breathing room, deprogram, and so on.





N: So if today's readers esteem mathematics they should see why we think in terms of teaching and educate. We have to resist or philosophy ridiculous! continues to be, effect of a true education or at any rate circumscribed. anything to convince you of the worry not, there is an easy way to get rid of these bad mathematicians or philosophers – one only has to cease paying them.



just as much as I praise my own field, have to oppose those who thereby pretend to make mathematics this imitation the lasting will be destroyed, Teachers will of course do importance of what they do. But

\*Authors' note: This last part is a bit rough (but which part? There seems to be three textual possibilities). It is however an almost literal quote of what Nietzsche (1872, 1874) wrote in his texts about the future of our educational institutions, and on Schopenhauer as educator. Our point of view is slightly different, as the reader will note in the Coda. But let's not go too fast! (Though YES, of course, you can peek at the Coda if you want!)

## Addendum

JEROME: Not bad! You managed to bring in those two guys, I like it. But I have a few comments.

JF: Go ahead.

JEROME: Maybe we should emphasize more strongly Dickens's (1854/1966) parody of a teacher who focuses only on facts "and nothing else". It provides a most needed comic relief while also getting right to the point about possible ways of depicting "teaching".

JF: Humm... I see your point, ok. But if we are going to do that, we might as well go and get more of those comic things, these caricatures you know, like in Molière or Ionesco...

JEROME: Or Plato's Meno! What a mockery he did there!

JF: Aren't you the one who always says we learn more about news from the newspaper caricatures than from the actual newscast? It also makes me think of Skinner when he wrote that "education is what survives when what has been learned has been forgotten".

JEROME: Funny. I also have this from Duckworth (1987): "By 'teacher' I mean someone who engages learners, who seeks to involve each person wholly – mind, sense of self, sense of humor, range of interests, interactions with other people – in learning". It fits well with Nietzsche's idea of joy, and our comment that the teacher be a part of that enjoyment.

JF: Wait wait wait. Now I'm not sure about all this... It's way too rooted in the teaching-learning paradigm. If we want to use those kind of ideas, we have to rework the whole paper! This would work if we were *slowly* moving away from the paradigm, using ideas that invite us, push us in that direction. It is doable, but that's not how I saw this article, not at all.

JEROME: But as you (or I?) said before, it's important to be inviting, to build on other people's work. We want to be engaging, not just disruptive and trash everybody and everything.

JF: Agreed, but the storyline I have in mind *starts from a rupture*, like those opening scenes in Hollywood movies. We are right in the middle of the action from the very first moment. And it is only afterwards that we sit back to figure out what is going on and why, etc.

JEROME: OK. Works for me. I'm not much into dwelling on the past, as you well know...

JF: Yeah... I mean, I want this article to be in the “no-teaching-and-learning land”, not somewhere in between. Yes, we carry on others’ insights, but I don’t see how we could use Duckworth’s quote, for example. It refers too clearly to teaching and learning.

JEROME: But another thing I like is how Duckworth talks about adults and children doing things together, sharing ideas, problems, solutions... Maybe that is what she means by “the having of wonderful ideas”? (But I amusedly confess not knowing, as I haven’t read her book!) Isn’t that a big part of what we have in mind? We move away from teaching and learning because it resonates with a fixed curriculum, a static vision of mathematics, a deficit view of people’s performances...but, at the same time, why insist on this? Surely nobody still thinks like that anymore! We are in 2018, after all!

JF: Going slow would be the usual way to explore these ideas. But we also have “disorder” here as a thematic for this Special Issue. I think we should take advantage of that fact to question the way we work with research papers as well. Everybody is quoting one another almost as if writing was a transparent business, just a carrier of ideas!

JEROME: We talked about that before in our book review for *Mathematical Thinking and Learning* (Proulx & Maheux, 2016).

JF: There is a whole sociology of quoting out there that we keep ignoring! We know that we essentially quote our friends and our enemies, and that putting other people’s words in your paper is at the same time putting your words in their mouths! I’d like to emphasize that.

JEROME: So how do you plan to do it? Graff has a whole section in his book on bad quoting practices. Are you going to take a leaf out of his work? Surely you don’t want to pull off a Sokal hoax...

JF: Well, yes and no. I think now is the time to turn to our Mathematics Education Digest on the other page.

JEROME: Ah ah! This one is hilarious! It is an interesting illustration of what productive quoting looks like... and how we can use it to generate new ideas!

JF: Let’s hope the reader keeps in mind that this is a joke (or is it?). And is thus both plausible and impossible. A mixture of familiarity and incongruity, indeed. But explaining it would kill the joke (or not)!

Authors’ note: At this point in the article, we are happy to take advantage of the space left here (the Digest needs a page of its own) to express our gratefulness for editors and reviewers who made the publication of this opus possible. Writing this paragraph “after the fact” has the particularity of allowing us to also include some of the comments that were made by the reviewers. We are particularly obliged to J.A. (who dared to disclose his name to us!). After admitting that he almost gave up a few times on finishing to read the article, J.A. goes on to share some very insightful questions and observations. Trying to deepen his understanding of our “doing|mathematics”, he wrote (more or less):

J.A.: You know... Halliday (1985) considers “doing” as a way of representing our experience in the world. Now, how do *you* “separate” such experiences from “doing” when you say we “make mathematics happen”? Are you suggesting that “doing” is a mixture of the “present” moment in doing and “previous” experiences? How do you distinguish between “doing” as representing and doing as producing?

This is a rich question to explore, and we have a remarkable answer to it. However, should we say that this margin is too small to contain it? Let’s just say that “doing|mathematics” *does* have a strong, undisclosed, relationship with time, and what we call “traces”. Both doing and experiencing draw on and create traces. Mathematics is like a becoming quality of these. Of course, we consider here a non-linear approach to time, in part inspired by Deleuze’s (1994) *Difference and Repetition* as well as Varela’s *creative circle*. So, as you can see, this margin is indeed too small!

Both reviewers also commented, in their ways, on the idea of changing referent for mathematics, relative to the Coda. Do we need a reference at all, they somehow asked? Right on! This is one of the subtle points we want to make without making it explicitly! But then again, maybe not having a reference becomes another sort of reference...



# THE MATHEMATICS EDUCATION DIGEST

## Chocking revelation!

A recent investigation revealed that Freudenthal's conceptualization of mathematics is centred on mathematical activity in its *hic et nunc*, and not on things being states to be acquired or developed! He in fact stated that working on problems, organizing ideas,

## Papert admits it!

Pressed by colleagues, Papert confess that being a mathematician is no more definable as 'knowing' a set of mathematical facts than being a poet is definable as knowing a set of linguistic facts, and that being a mathematician, again like being a poet, or a composer or an engineer, means doing, rather than knowing or understanding. He then stressed the importance of developing MWOT, that is, mathematical working on things.

## Time to move!

If we are serious about rejecting the platonist view of mathematics, it is time to finally "shift away from emphasizing mathematical activity as being to do with converging to pre-defined and well-known concepts" says Tony Brown in a 1994 book chapter. He continues noting that here "the emphasis is not on understanding a particular concept but is more concerned with the process of understanding."

## Guy Agrees, who's next?

One of the famous founders of French Didactique des Mathématiques, Guy Brousseau, took stand today saying that teaching is "impossible". He detailed the paradoxes arising from wanting students to understand something without imposing on them the teachers' rhetoric. He

## Caught cheating!

The Lockhart report outlines fraudulent abuse of mathematics with students, making the highly creative discipline a boredom: "Students learn that mathematics is not something you do, but something that is done to you". On top of his report, he implores us to stop imposing mathematics on students as concepts and methods to be learned, and rather allow them to simply experience the free-spirited (joyous) 'pairing' of doing mathematics. "Play games, do puzzles, don't worry about notation and

## What Agassi had to say

Karl Popper's student J. Agassi says he wants to see destroyed the system in which adults "motivate" children to learn or discover mathematics, using textbooks and the like. Instead of lying and training for dependence, he said, we should engage with kids as fully responsible, real mathematics researchers from the very start, take them as active participants in

## Yes she did!

Nearly 10 years after the publication of her now apparently forgotten groundbreaking publications on using errors in mathematics education, Raffaella BORASI did it again! Her plot against the view and use of so called mathematical "errors" as diagnosis-remediation tools pictures mathematics as the result of a process of inquiry in which uncertainty, conflicts, vagueness and the like are essential. She then urges us to embrace them with students, while embracing the CREATION OF MATHEMATICS.

## PROTESTS IN MATHLAND

Many mathematics education researchers erected barriers on School street in Mathland today. New slogans were shouted to resist the "teaching-content-learning-content" endemic that has clogged and suffocated mathematics education for a number of years. Out of these hooligans were heard the now-famous no-no slogans like "no problem, no maths", "no errors, no maths" or "no question, no maths".

## Fatal accident

Mathematicians using the (in)famous Moore method in which "the students is best taught who is told the least" accidentally killed teaching today. By way of requiring students to solve theorems and develop proofs while being forbidden to use any textbooks or refer to resources of any kind, they placed them in an environment so conducive of mathematical action

## A breath-taking duet

Stephen Brown and Marion Walter reiterate, reedding the famous opus in which they picture students doing mathematics "for the fun of it". They showcase "questioning" as what mathematical activity in its truest sense should be about, scoring high on exploring, wonderings, and musings in mathematics for its own sake. "Turn mathematics upside down",

## Stop it, NOW!

John Bennett took advantage of a TEDx talk today advocating we should stop teaching math in school. The math teacher termed math instruction unnecessary, and went on arguing that math should only be offered

Us...

*A runaway train races toward five people tied to the tracks. You just find yourself next to a lever: If you pull it, the train will switch to a different track where only one person will die. What do you do?*

Both Nietzsche and Derrida help us think outside of an educational ideology driven by pre-determined (social) functions and the promotion of the economy (e.g., Higgs 1998). Reading Derrida, Higgs (2002) continues:

The philosophical challenge of re-thinking education, of deconstructing education, does not consist in changing, replacing, or abandoning education. On the contrary, to deconstruct is first and foremost to undo a construction with infinite patience, to take apart a system in order to understand all its mechanisms, to exhibit all its foundations, and to reconstruct on new bases.

JF: I don't agree with all that, especially the mechanical rendering. But there is something there. And the point is precisely that there is "something" there that we don't need to call "teaching and learning". Something that is happening anyways – let's call it "education" – but a form of education which is terribly hindered by an instructional paradigm that still dominates both our actions and ways of thinking. Jim Neyland (2001) really spotted that beautifully in his "ethical critique", remember?

JEROME: Yes, but let's stay with Nietzsche and Derrida for a second.

JF: In terms of deconstruction, the case I wanted to make here is that education is about "leading out" not "taking somewhere" – and especially not about taking others to the production of pre-determined, recognizable production of mathematical ideas. Mathematics *is* the way by which I offer to *educate* the other, those children.

JEROME: You mean doing mathematics, not mathematics as a thing in itself!

JF: Right. And now I am wondering: how do we explain why we do not use the good old "guide" metaphor, for example?

JEROME: I wrote somewhere about "objectives to work on" as opposed to "objectives to attain". Maybe we could use that?

JF: Could be a start. Now that I think of it, "guiding" for me gives the impression that there is something out there I want to *show* you. Consider the word itself: from the Frankish *witan*, or the verb "wit", meaning "to know" in its archaic form. *Witan* itself derives from the pseudo-indo-european *weid-* meaning "to see", or "to find". I am still a teacher showing the way, you are a learner seeing new things. But from my point of view, there is nothing to be taught or to learn, no externality.

JEROME: Going to the roots of words is not the way to do it, I think. We could do the same thing with "teaching" and "learning" and find *interesting* roots! It's a catch-22 situation. Let's stick with ideas, no need to go to the roots.

JF: You are clear on that! But remember, this is essentially *playing* with roots, so catch me if you can!

JEROME: Sure, sure. But remember, the concept of "catch me if you can" is indeed about aiming to be (un-)caught... Now back to what we were saying. We should emphasize that there is not even "a way" to be shown in an objective sense, no "mathematical way" of doing or thinking that should be demonstrated, or made accessible. We also have in mind Standish's (2001) observations that the very condition for education is one where students *always* go beyond what could be fixed

meanings, curriculum objectives, the authority of the teacher, and so on. Sarrazy (2015) discusses the same in terms of a necessary “transgression”, where students need to go beyond what they are told.

And Sarrazy’s paper  
is in *Chroniques*,  
our new online  
research journal!!

JF: Exact, because accepting authority supposes the pre-existence of something relatively static (those mathematical ways), and some sort of support for it. (Everyone will agree it is not in the books!) But this is problematic for some reason. So for us it is about *doing* things together, doing mathematics. Making it happen, and by doing so, offering people various (mathematical) experiences.

JEROME: Even more importantly, remember our doing|mathematics; “an expression that alludes to the emergent made-up nature of both the doing and the mathematics and their dialectical relationship in lived-life mathematical experiences” (Maheux & Proulx 2015, p. 214).

JF: Yes!! One key point we don’t highlight enough is that this is mostly a commentary on human experience and on the nature of mathematics. It is a perspective on mathematics strongly influenced by humanistic writings...

JEROME: ...and a perspective on human experience deeply connected to the “enactivist” literature.

JF: You like to say there is no mathematics outside of the actual activity of people doing mathematics.

JEROME: And “Doing|mathematics is both doing something (some thing) recognizable as mathematics, but also producing mathematics as this thing that we are doing when we do what we do. Although the two interpretations of the expression doing|mathematics are incompatible as fixed states, they echo the productive circularity at the heart of our enactivist thinking, according to which there is no objective end or start, but only an observational beginning. Both directions thus develop simultaneously, nourishing one another and drifting in the same current.” (p. 215)

JF: With Maturana and Varela for example: All doing is being, is knowing, in the bringing forth of a world through our coordination with ourselves and our environment... [a confusing sentence].

JEROME: So what we say is that even in a situation that people would normally describe as a teacher teaching and a learner learning mathematics, what *we* see is in fact a particular case of doing|mathematics, the offering of a particular mathematical experience. And, probably the other way around: people see teaching and learning in many, if not all, situations that we would call doing|mathematics.

JF: If we recognize it as such, yes. But people will also want us to tell them how thinking about these situations in the way we do is fruitful. Especially since the setup with Nietzsche and Derrida is so provocative (maybe even offensive).

JEROME: Is it just a matter of vocabulary? Why not keep calling “teaching and learning” the forms of doing|mathematics taking place in school? Does it really open to new possibilities, or make some (maybe less desirable) possibilities more unlikely?

JF: It is difficult for me to forget Heidegger’s observation that we don’t simply think with language, but that language “thinks us” too. There is a very strong argument to make in relation to whatever role content or curriculum are assumed or required to play in relation to doing mathematics in school. Thinking in terms of teaching and learning *alludes* to the belief in some sort of transcendental existence of mathematical ideas, concepts, etc. But deconstructing the role of the teacher (and the student) quickly reveals how this can hardly be the case. Trying to meet that requirement is not only futile, but also very hurtful.

JEROME: This is where ethics comes into play, where we take on Cahen’s (2001) questions around educating the other as other, and Higgs’s (2002) reflection on the nature of an educational encounter in that context. This is also the point where we can consider the ways in which ethics still leave room for a conception of education based on a “disruption” of the other.



JF: What we wrote about Levinas and his idea of welcoming the other without reducing his/her otherness to what we know, but instead be-with and do-with the other... conceived of mathematics as an (ethical) occasion to encounter otherness. Neyland's work I mentioned...

JEROME: So we move away from thinking in terms of teaching and learning because we see the value of letting go of curricular imperative and an objectivist-transcendental view of mathematics and mathematical activity. One "benefit" is that it helps us see violence at the heart of the kind of mathematical experience offered when doing|mathematics is organized as if someone is teaching to someone who is learning.

JF: Thinking in terms of doing|mathematics tells us that we can still envision something like "mathematics education" outside of this instructional or directional paradigm. We can think of offering mathematical experiences that do not assume the impossible erasure of the teacher, but rather hinges on the ethical presence (*praesent*- 'being at hand', from *prae* 'before' + *esse* 'be') and encounter (Latin *in*- 'in' + *contra* 'against') of the other. By doing-with one another...

JEROME: Again your attraction to roots! Maybe we need to illustrate what we mean here, give some example of what these kinds of mathematical experiences could look like in school... and acknowledge the institutional contradictions one might see in relation to these opportunities.

JF: Examples are easy to find. I will let you give some.

JEROME: The first thing that comes to mind is the way Borasi (1996) worked with the idea of "errors" as springboards for inquiry. She uses the word "error" but makes it clear that what she is talking about is much broader, closer to what you call "mathematical imperfections": curious mathematical phenomena, questionable conventions, intriguing ambiguities...

JF: Like the way we add ratios versus fractions, or how  $x^2+y^2=c^2$  can describe both a circle and Pythagoras' theorem, or the evaluation of  $0^0$ , and so on.

JEROME: ...and then explore them with students, often not knowing where such exploration could lead. But that's not a problem, in part because students are not left alone. The experience is about exploring these phenomena together, it is about working on (again my objectives 'to work on' vs 'to attain'). Borasi's analyses show us precisely how playing with such ideas can be mathematically enriching.

JF: I made a similar argument in favor of experimenting with various "by-hand algorithms". There is absolutely no reason to "learn" how to extract a square root by hand, but it can be a fun and deeply enriching mathematical thing to do, and it can open up to a rich field of questions.

JEROME: I sense some Brown and Walter's (2005) problem-posing here...

JF: As I see it, your work on mental mathematics also illustrates this. You ask people to work on an equation or a geometrical problem without pencil and paper, and you create an opportunity for them to articulate all sorts of mathematical thinking, to explore their limits and potential...

JEROME: Mathematical activity in its fullest sense: that of making sense, mathematically! And, there is nothing there that I am "teaching", nothing for them to "learn" ... yet people attending such sessions call that "great teaching"!

JF: And I guess the *liberation* part is right there, in the going free from the impossible obligations: to teach, to impel the other to learn. Mathematical activity is imprisoned in that paradigm, assessment being the jailor-in-chief. In a classical form of the Stockholm syndrome, we feel that we need these jails and guards to survive. What would we do without them?!

JEROME: I have come to a point where I actually don't see the benefit of thinking in terms of teaching and learning. When you start orienting yourself towards problem solving and mathematical activity, rather than transmittable or discoverable concepts, the distinction is simply not helpful anymore.

JF: And all the fallacies that surrounds these ideas quickly become unbearable! What is knowledge? What is an ability? How are either acquired? Where are they stored? How are they transmitted? Of course, these are all allegories, nobody *literally* means that there are things such as knowledge, acquisition, construction, discovery. These are just figure of speech, fictions.

JEROME: Until they are not useful anymore! And that's one of our points: they are not useful! So letting go of them should not be that hard... unless we end up convincing ourselves that those fairytales have some sort of objective existence.

JF: But the question of the “applicability”, about changing school or what people are doing... that is a more delicate issue. This is why we wanted this paper not only to be about disturbing the teaching-learning side of mathematics education, but also talk about mathematics education research, and how we see our role and our ideas, in relation to such quest.

JEROME: Time to change section.

*An anti-math terrorist group took your research participants as hostages. They will kill a selected five unless you tell them who is the best at mathematics in the remainder, so they can kill only that person. What do you do?*

## **Towards (the future of) doing|mathematics: a game book**

*...The shift from a stance of reform to a stance of evolution does not exclude active intervention, but the role of the change agent becomes less like the architect or builder and more like the plant-or animal breeder whose interventions take the form of influencing processes that have their own dynamic.*

(Papert 1997, p. 418)

P0. Some readers might be doubtful regarding the shift Papert suggests here. Why not go with the flow and merely try to improve what is done in schools – or at least offer teachers some realistic possibilities? Of course, thinking in terms of teaching and learning is not just being delusional! If you think that it makes little to no sense to believe that these kinds of ideas can have any impact on what is going on in schools, proceed to P8. If you don't think that this is an issue, turn to P6. (And don't forget to keep track of your scores and personal notes!)

P1. When we say “the future” and have in mind changing schools or developing research, we often implicitly think in terms of a period of time following the present moment, something “still to come”. Moreover, it seems natural to think about that something in terms of “events”, and all the actions and circumstances constituting them. Playing a bit on words, we can conceptualize the notions of “future” and “events” quite differently. The word future comes from the Latin irregular suppletive future participle of the verb *esse*, “to be”. The same verb is at the origin of “present”, made of *prae-*, “before, in front”, and *esse*, which is of course the core root of the word “essence”. So thinking about “the future” of mathematics education, of schools, and that of our own research activity is in fact thinking about a modality of its being, of its essence. The “present” is always behind this modality. Deleuze discuss it in terms of difference and repetition. Husserl evokes it in a flash, saying that “geometry is on the way to its origin”. Coherently, we can playfully think of “event” in a similar sense. The word comes from the Latin *eventus*, a form of *evenire* made of *ex-*, “out” and *venire*, “to come”. Events are a coming out of the things that emerge from a particular way of being. And these ways of being, of course, emerge in return from those “coming to be”. The grapheme “doing|mathematics” we adopted s(t)imulate this circularity. So, in a nutshell, thinking about the future is thinking about the nature of mathematics, mathematics education, mathematical activity, and so on. If your hair, ears or eyes are hurting too much,

turn to P11. Otherwise, go to P9 to read what “always already” alludes to, or jump to P14.

P2. Traces are amazing. Seriously. We have a marvelous demonstration of this proposition, but maybe some other time. However, since research is for us fundamentally about stimulating more re-search, simply mentioning it together with a few names like Châtelet, Derrida (again) and (or course) Heidegger should suffice to let you suppose that the proof is, indeed, “out there”, only waiting to be collated... if you dare do it. Let’s just conclude this adventure in which YOU are the hero by saying that traces are what make the impossible possible. Traces, in our view, do not signify, refer, or represent: they do not simulate some external, pre-existing ideas. Traces *stimulate*. Our desire is not to affect schools, but to encourage others, YOU, to think (with us), to examine, question, reflect on, challenge, “deconstruct” the nature/being of mathematical activity in school. We hope to encourage you to take on the issue and keep it going. And thereby, *maybe*, to make more likely the impossible evolution (Latin *evolutio*, “unrolling”) of doing mathematics in school outside of the teaching-learning paradigm. After all, as Derrida observed, only the impossible is *possible*. Now, your quest has come to its conclusion. You rise as phoenix, stronger than ever (add 10 to your strength, wisdom and endurance). Ready for your next adventure. The End (towards a new beginning, a future...).

P3. A few years ago, we developed a proposal, as we have in this paper. The idea was to stop thinking in terms of mathematical knowledge, and focus instead on mathematical actions, mathematical doings. Many arguments were raised: (a) ethical ones, (b) epistemological and ontological ones, and obviously (c) mathematical ones! Summarizing these ideas in a paragraph is impossible. If you need help distinguishing between enactivism, constructivism, or need to figure out what the heck epistemology is, go to P4. If you are comfortable with these ideas, proceed to P5. If all this seems too much, turn to P11.

P4. There’s no good place to start on this, so we will play it hard, and give you some of our favorite references. You may have to get lost in the woods before getting back... For constructivism, we recommend von Glasersfeld’s book *Radical Constructivism* (1995). For epistemology, our classic is Bachelard (1937) *La formation de l’esprit scientifique* (yes, in French...originals are often better...). For enactivism, *Tree of knowledge* (Maturana & Varela, 1987), *Embodied mind* (Varela, Thompson & Rosch, 1991), and *Ethical know-how* (Varela, 1999) are considered “classics” (however, we have argued for long that there is no official literature for enactivism – see our short research note in Proulx, 2016. We also produced a series of papers on enactivism and constructivism, but we will let you find them!). If you are ready to go on, turn to P5. However, if after (really) reading (all) the above, you still believe that knowing happens in the head, go to P11. On the other hand, if you honestly just want to quickly get a sense of what this is all about, fly to P13.

P5. You have a good sense of why we are doing this. Here is another way to put it: Châtelet (2000) makes a powerful distinction between “the possible” and what he calls “the virtual”. He explains that while developing the possibilities coming out of a given state of affairs is an important endeavor, the New can only arise from going beyond the possible and into the virtual. Doing this requires taking “traces” (like words, ideas) in unintended ways, attending to the excess meaning they inevitably freight, and from there opening a new space of possibilities. He gives examples of this in the history of mathematics. Maybe part of what we do here is similar to that. Words like teaching and learning, and the ideas that come with them, serve to keep us thinking about mathematics education beyond what seems possible. If you are not sure about the relevance of this, turn to P6. Otherwise, you can carry on to P8.

P6. It might be useful to recall some of the conversations in which scholars before us expressed the crucial difference between doing research and wanting to “fix” mathematics education. A good start is Bauersfeld’s (1977) assertion that “research outcomes have influenced the reality of mathematics



instruction and mathematical learning on a very small scale only”, which he blames on “[following] the need of school practice rather than hurrying on ahead”. Slightly more positive, Kilpatrick’s (1981) celebrates the *reasonable ineffectiveness of research in mathematics education*. Through deconstructing the opposition of basic-fundamental versus applied-pragmatic research, he argues that the dichotomy is in the eye of the observer, and goes on saying:

Too many mathematics educators have the wrong idea about research. They give most of their attention to the results. They think it is primarily important for teachers to know the results of the research on a given topic. They give a high priority to summarizing and disseminating research results so that teachers can understand them. In a nontrivial sense, however, the results are the least important aspect of a research study [...] A researcher makes a contribution to our field by providing us with alternative constructs to work with that illuminate our world in a new way, and not simply by piling up a mass of data and results. (p. 27)

In short, research does not have to be geared toward finding answers to the so-called problems of practice. One of its role is the generation of ideas, from which we can, following St-Exupéry (1948/2000), *mettre des forces en mouvement*. If you now understand why we play with/against words like teaching and learning, go to P8. If you are still unsure why we want to do that, go to P12. But, if this remains somehow offending, nonsensical or pure fluff, proceed to P11.

P7. One important question we often forget to ask (ourselves) is: how do we (or our colleagues) actually read research papers? It should be clear by now that one of the points we are making throughout this article is that we can gain from being a little bit more creative in the way we make research available to one another. The fragmented, conversational, evolving, colloquial, diverse – and, at the same time, obscure, humorous, or aggressive aspects of this article wish to mirror how we (Jerome, JF) generally read research (and make something of it). And the inclusion of this comment here in the game book is not insignificant! Why? Because unless you are cheating in the game – which is clearly not a bad thing in this case – you are not supposed to read this! Bluntly said: we always write more in a paper than we want people to read, and we always read more in a paper than what the authors wanted us to read. Doing it this way is a little grotesque, but what is not? (See Bakhtin, etc.) And the point is actually an important one (partially hidden, but in plain sight!) in relation to “the future”. We’ve been writing about “doing|mathematics” for some time now, and writing is as much for the future as it is for the present. Who knows what ideas will become once you send them out into the world, or what people will make of them? (Including us!) Writing now about our interest in freeing mathematical activity from teaching and learning categories IS creating a future in a very practical, yet completely undetermined way. We look forward to it. At this point, go back to P0 if you want to play by the rules. Or, continue to cheat on toward P8 if you were reading the paragraphs in their order of appearance. If these options do not suit you, go back to P7 to recursively stand-still... and re-enjoy!

P8. Now the question is: how does these new possibilities connect with Papert’s idea of “influencing processes that have their own dynamic” in the quote opening this section? (Some background on Papert’s incredible propositions for mathematics education are outlined in P15). We shall first question what we mean by “the future” of mathematics education, and then argue how what we foresee is always already there while at the same time show how embracing these ideas brings us beyond what is possible. To explore “the future” go to P1. If the notion of future as “being” makes sense to you, skip to P9, unless you are familiar with the formulation “always already”, in which case you can move to P14. If you feel lost and ready to give up, you can still try P14, but you’d probably better go straight to P11.

P9. The expression “always already” calls attention to the changing nature of events, of what comes to be. Being, like doing|mathematics for example, is necessarily ahead of itself: it anticipates the possibility for what comes to be – anticipating both that our doings will become mathematical, and that

mathematics will be what we do. But this anticipation also affects being backward, since all that comes to be seems to be preceded in its eventness, e.g., as a possibility, or to be recognized as such (Bakhtin terms this the eventness of being). What we call the future is always already present but also always already absent (i.e., away from being, from *ab-* ‘away’ and *esse*). Doing|mathematics is this movement, the limitless wandering beyond and between being and non-being, present and future. Oversimplifying, let’s say that when we do mathematics, what we do is never the whole of mathematics, and that mathematics is never the whole of what we do, although all of mathematics and doing in each single event is the realization of the essence of doing mathematics, of its coming to be. Certainly still quite cryptic and mysteriously circular, this should nevertheless be enough to get you through P14 and the rest of this adventure. Well done!

P10. On movement. In short, enactivism leads us to see mathematical concepts as action, and not as things or entities that one can grasp. Mathematical concepts are thus seen as breakpoints in the mathematical activity, in its enactment: reifications of mathematical processes as Sfard would have it, or crystallisations for Tall or encapsulations for Dubinsky. Heraclite apparently said: Nothing permanent but change. This marks the paradoxical and dialectical nature of movement and constancy: movement needs constancy, and vice-versa. For us, mathematical concepts are themselves traces of an ongoing mathematical flow of permanent change. Its consistency is in what mathematics becomes. Their perceived fixity (an effect of retinal persistence) is at best a useful fiction, when traces get the activity going, like still pictures from which we recreate movement (see, e.g., Roth & Maheux, 2015). In the moment-to-moment of doing mathematics, we are also observers (as Maturana puts it) whose commentary (a difference that makes a difference, in Bateson’s words) of ongoing activity may cause mathematical concepts to emerge in a similar way (for oneself or another). But, as John Stuart Mill so well expressed it, our “tendency has always been strong to believe that whatever received a name must be an entity or being, having an independent existence of its own”. If you are not sure why that would be a problem or if you want more, go (back?) to P4 (and read the stuff this time!). If you think your brain is melting from so much nonsense, go to P11. If you still have some strength in you (roll the dice, just to make sure!), go to P5.

P11. At this stage of your adventure, you realize the battle was fast and furious, and in the end you lost. At your feet you observe an immense precipice dropping away below you. And just as you rise unsteadily, you feel a great blow to your back. You are sent flying forward, falling into the everlasting darkness of mathematical death, in a Lockhartian sense (2009). The End (of ?).

P12. So where is this going? Looking back into how *we* came to this perspective might help make some progress on that, but here again, let’s not suggest that there is some kind of planned unfolding. We genuinely don’t know where this is going... and that’s why we can research it! If you haven’t read our papers on “doing|mathematics”, continue to P3. If you know that stuff, jump to P5, or go straight to P8 if you’ve been there already.

P13. Learning has often been defined as a transformation in a state of being of a learner: s/he couldn’t do this before, and now s/he can. The transformation might come from “acquisition” (e.g. of new knowledge), result from an adaptive process (e.g., in the Piagetian tradition), from the appearing of new linkage (e.g., when psychologists talk about transfer), and so on. A radically different way to see “learning” is in terms of “an ongoing, recursive, elaborative process, not an accumulative one” (Davis, 2004, p. 130). For Maturana, this process is an integral part of living, inseparable from all other biological or psychological processes. Or, to put it better: it actually only exists in its ongoing distinction (from all other processes) on the basis of some (specific but variable) criteria. So Maturana (1988) writes: “learning is a commentary that an observer makes about two moments in the *epigenesis* of an

organism in which he or she does not see the historical process that connects them” (p. 58). There is no learning lying “out there” to be seen, independent of the observation. The students’ “learning” belongs to, is a property of, *the observation*, and only lives there, in the observing, in the ongoing commentary made from observing the mathematical activity of the student (see, e.g., Maheux, Roth & Thom, 2010, for some early thoughts on this in the context of children’s mathematical activity). If you are eager to learn some more, by all means go to P10! If you had enough, go back to P4. If you feel you’ve been patient enough with us and are now ready to move on from this paper, slide to P11.

P14. With these ideas in background, it should be easier to appreciate why we say that the perspective we advocate in this article does not call for reform. However, we certainly do not suggest the preservation of any sort of status quo, let alone the “improvement” of mathematics education as it currently takes place in schools. For one, we think outside of reform and improvement because they are themselves rooted in the teaching and learning paradigm. Working in terms of doing|mathematics, of mathematical activity, is a shift in stance. Such thinking looks at what takes place in schools (and elsewhere) in a different way. And it starts by acknowledging that doing|mathematics is always already there, while at the same time still “yet to come”. Teachers and students already do mathematics, but how they do it is shaping what doing|mathematics can signify. This mode of thinking repeatedly questions what we mean by doing|mathematics, what these concepts and processes are all about, why and when they contribute to different ways of being in the world, and asks, moreover: where is mathematics? And who is it for? We are saying that mathematics education does not need to be conceived in terms of teaching and learning, and that “teachers” and “learners” do not need those labels. We can see them simply as people doing|mathematics. These kinds of mathematical activities not only “have their own dynamic”: they realize some of the possibilities, they incarnate in various forms the “essence” of doing|mathematics. An important part of our work hopes to elicit this, leaving traces (e.g., words) from which to consider various mathematical experiences. To put it differently, “toning down” (taming?) mathematical activity in schools, envisioning it as “only” another instance of doing|mathematics, makes it easier to imagine a larger number of possibilities. Perhaps even the impossible. If you’ve finally had enough, read P11 now. To take one more step into the unthinkable, turn to P2.

P15. Seymour Papert was a mathematician with a vision of what mathematics is all about. Placing the mathematical activity in the center of his reflection, he strongly critiqued mathematics as it is practiced in schools. He said, for example: “there is a big distinction between something that I love and I call mathematics and something called ‘math’, which is what we teach in schools”, and stressed the dullness of “discovering something the teacher decided you got to discover”. Advocating that children should pursue their own mathematical interests, he explained: “Being a mathematician is no more definable as knowing a set of mathematical facts than being a poet is definable as knowing a set of linguistic facts. [...] being a mathematician, again like a poet, or a composer, or an engineer, means *doing* rather than knowing or understanding” (1972, p. 249). In doing mathematics, children develop mathematical ways of thinking, of being mathematical. As a visionary, Papert saw in the computer an incredible tool to aid students in the pursuit of doing, for example, by programming in a mathematically oriented environment such as Logo. But again he made it clear: “It is not true to say that the image of a child’s relationship with a computer I shall develop here goes far beyond what is common in today’s schools. My image does not go beyond: it goes in the opposite direction” (1981, p. 5). In that sense, our assertions in this paper are pale in comparison with what Papert asserted and envisioned. You can now return to P8.

Author’s note: The authors also want to thank the “second” reviewer for the delightful “suggestion” that we replace the Sheffer stroke we use in “doing|mathematics” with an exclamation mark: “doing!mathematics”. This is clearly something we consider doing in the future. And, it is so much easier to find on the keyboard!

## Coda: The last stand

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**Abstract** In a recent article, Maheux and Proulx (2018) QUESTIONED the focus on teaching and learning mathematics in schools, SOMEHOW arguing in favor of a more genuine view of mathematics as an activity. In doing so, they grounded the idea in the authentic and ACTUAL PRACTICE of people doing mathematics. But, in so doing, they left implicit and unquestioned one important assumption: that this “actual practice” SHOULD refer to how mathematicians do their work. In this article, we bluntly ask: *Who says that mathematicians need to be the reference point when it comes to mathematics?*

**Keywords** Mathematics • Education • (Research) • Liberated • Teaching • Learning • Towards • Future • Doing|mathematics

### 1 Throwing some unfinished/able thoughts in the way

In a recent article, Maheux and Proulx (2018) questioned the teaching and learning METAPHORS for mathematics in schools. Calling for what would be a more genuine view of mathematics as an activity, they seem, however, to assume that it is the professional mathematicians’ work that should serve as a REFERENCE to define such genuine mathematical activity. This looks to us like an unfinished thought. In a slightly dishonest way, we use the opportunity given by the editors to respond to this implicit propo-


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Published online: 01 January 2018

 Fingers

sition and push these authors' thinking a few steps forward. We would like the reader to see this response as a turn in a sort of conversation about genuine mathematical activity.

## 2 Back to School

If you think about it, school is THE setting where MOST OF mathematics is done in this world. Professional mathematicians represent a very tiny portion of all the people who do mathematics<sup>1</sup>, and what they do is relevant to even FEWER people! As Davis and Hersch (1982) put it:

The ideal mathematician's work is intelligible only to a small group of specialists, numbering a few dozen or at most a few hundred. This group has existed only for a few decades, and there is every possibility that it may become extinct in another few decades. (p. 178)

This needs to be disturbing, we claim. Another way to see how tenuous is the connection between the mathematical ideas we work on in schools and professional mathematicians' work is in the observation that most of those school-based concepts are HUNDREDS OF YEARS old (at least 2.5k in the case of the Pythagoras theorem) and most if not all of those, in the way we work at them in schools, do not play a role in professional mathematician's work. We even hear of mathematical concepts that were invented almost entirely for the school context: Bronner's (1997) notion of *i-decimal* numbers is one; how we now think in terms of slope and its "effect" on Cartesian representations might be another. Those appear to be good examples of what Brousseau (1998) refers to as "les MATHÉMATIQUES DE LA DIDACTIQUE" (see more on these ideas in Proulx, 2012). A milder, nevertheless powerful observation in that sense is in Agassi's (1980) remark that:

The mathematics required by (1) the amateur, (2) the applied mathematician, (3) the mathematics teacher, and (4) the research mathematician, are so very different that each needs a different agenda [even] when all four want to know what an axiom is [...] (p. 30).

Let's play a little. We could argue that if we were to STOP teaching mathematics in schools, chances are mathematics would not continue to "live" very long in our society. Isn't this what happened with Latin, for example? Oho! Interestingly, or so we think, this observation also gives grounds for challenge the common assertion that "the GROWTH of mathematics is a result of the creativity of individual mathematicians" (Henley, 1992, p. 40). Just as it can be argued that language truly evolved through its EVERYDAY USE (including how teenagers create new words and meanings), we may be able to posit that the BEATING HEART of mathematics is not in the mathematician's work. (We'll make enemies here, but let's keep them close, right?) Couldn't it be that it grew naturally out of the mathematics done daily in school – maybe even from schoolchildren's mathematics, as Les Steffe called it?

Would it follow that mathematical classrooms should be taken as THE fundamental creative/productive environment for mathematics, and thus as the referent when it comes to thinking about DOING|MATHEMATICS? This is a crucial question, especially if it is the mathematical activity itself that is of interest. In that sense, the proposed move from content to doing would also invite movement towards a new referent: that is, from mathematicians' offices and blackboards (!) to schools' classrooms and whiteboards (or TBIs!). Such a movement leads US to wonder if we hadn't got it wrong from the

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<sup>1</sup> In his book on the history of mathematics, Escofier (2016) estimates at 100 000 the number of mathematicians on Earth. Any one single major city counts more students doing mathematics in its schools!

beginning... Listen to this: maybe ACADEMIC MATHEMATICS is, in fact, a form of APPLIED SCHOOL MATHEMATICS, and not the other way around! How fun is that? Though wild, such an assertion makes visceral sense. Ormell (1992) offers a COMPELLING ARGUMENT along that line, related to the New Math and the foundations movements: when the New Math entered schools, he explains, the basic concepts which organized teaching and learning objectives faced an “intensity of questioning which they had formerly been spared [... which] soon revealed the awful truth: that they were based principally on convenience, not on the fearless clarity everyone had previously supposed.” (p. 5). School mathematics provoked this SECOND CRISIS in the consistency of and confidence in mathematics. As the old idiom goes: the best way to learn about something is often to teach it. But let’s not plunge into *clichés*...

### 3 Looking for something new

Alright... we have to say: nothing in this is really new. Other scholars have questioned, implicitly or explicitly, the dominance of mathematicians’ work when it comes to considering school mathematics. Let’s just mention two. We think of Bauersfeld and his colleagues (e.g., Voigt, Krummheuer) when they take as “primary point of reference the local classroom microculture rather than the mathematical practices institutionalized by wider society” (Cobb & Bauersfeld, 1995, p. 9). Cooney (1988) somehow committed a similar “crime” in his observations about how mathematics is viewed. The general views about mathematics in society and in people using mathematics (mathematicians included) cannot change, he argues, until these changes happen IN SCHOOLS FIRST. We could find a number of similar points made about the nature of mathematics supportive of the idea, alas the narrow margin spaces...



Figure 1. Two kids

On the other hand, what if there is also, behind all this, a deep commitment to what Dewey emphasized when he said that “education is not preparation for life [but] LIFE ITSELF”? Some of our mentors were quite keen to tell us that school mathematics is not merely a preparation for some sort of “real math” done in the university, or the industry, or even at the supermarket. SCHOOL MATHEMATICS is mathematics itself – especially for school students, see Figure 1 – and could even be regarded as what other forms of mathematical activity “prepare for”, making it possible for the next generation to ENJOY mathematical experiences! Of course, it would be demeaning (pushing on ridiculous) to reduce these various mathematical activities in the same way we seem to reduce school mathematics – so that’s not



the point. And, as one reviewer of Maheux & Proulx's (2017) paper commented, how incoherent and non-inclusive would it be to reject forms of mathematical activity that are different than school ones. How disturbing or disorderly to take one hegemonic or dogmatic idea down and replace it with another one... Food for thought.

(But let us open a parenthesis within this parenthesis! JEROME recently read on the well-known work on open-problems in schools by Arsac, Germain and Mante (1988). In this BOOK, the authors suggest that students be encouraged to act as research mathematicians by working on CLASSICAL MATHEMATICAL PROBLEMS – so they can experience 'authentic' mathematical activity, as mathematicians do. If we buy the idea of taking school mathematics as a referent, a reaction to this body of work could be to ask: what are the classical mathematics problems IN SCHOOLS? What are SCHOOLS' CLASSICS? Now we are having fun! Hum... we can think of a couple of things (not all flattering), but let's just name one: mental calculations. Could this be a domain (of practices) that is essentially about/from/for school mathematics?)

But let's stop here for now. There is something rich and deep for us in the realization that ALL FORMS of mathematical doings presuppose each other, draw upon each other, nourish each other, precede and succeed EACH OTHER and even can "prepare", to a certain extent, each other by making it possible to move between, or in-between them. Here we have hit upon a way of thinking where describing what is done in one instance – in terms of teaching and learning – makes as little sense as employing these concepts to talk about mathematical activity outside of school. It is also an important move toward an expansion of our ways of developing mathematics, furthering its concepts and discipline. Something WE OWE... TO THE FUTURE. (Regardless of what that future is, see Figure 2).



**Figure 2.** Two more kids

## References

[Some are missing – yours to find. Or perhaps we have omitted them to force you to contact us and engage in discussions. Please do! And, through this, hopefully we'll find them for you...]

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